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G_{\max} for sand by bender elements at anisotropic stress states

L. Bødker

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Laboratory Testing Paper No 11



**GEOTECHNICAL ENGINEERING GROUP
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G_{max} for Sand by Bender Elements at Anisotropic Stress States

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SYNOPSIS: Analyses and design of foundations of structures exposed to dynamic loading requires knowledge of the dynamic properties of the soils. An important parameter used in the dynamic analyses is the maximum shear modulus, G_{max} . In the present study G_{max} is determined by means of piezoceramic bender elements for two types of sand and with void ratios varying from minimum to maximum. The tests performed are carried out in the Danish Triaxial Cell, and G_{max} are determined at different isotropic and anisotropic stress states. The main result of the test program is that G_{max} is primarily influenced by changes in the mean effective stress, p' , but also slightly influenced by applying shear stress. As expected the stiffness increases with decreasing void ratio.

1 INTRODUCTION

Analyses and design of structures exposed to dynamic loading such as oil drilling rigs, bridges, high speed railways etc. require knowledge of the dynamic properties of soils determined at low strain levels. The relevant properties for the analyses are the stiffness and damping of the soils.

The maximum shear modulus, termed G_{max} , is an important parameter for predicting the response and the behaviour of soils exposed to dynamic loading. The relevant strain level is very low, typically less than 10^{-3} %. A very simple and straightforward method for determining G_{max} is to perform tests using piezoceramic bender elements, Dyvik and Madshus (1985). Using a set of bender elements as transmitter and receiver a shear wave is propagated through a soil specimen. Based on the recorded travel time, travel length and density of the specimen the shear modulus, G_{max} , can be calculated.

The principle and validity of the method has been studied and some of the results are stated in Dyvik and Madshus (1985), Dyvik and Ol-

sen (1989) and Suoto et al (1994).

The main purpose of this work was to study the effects of anisotropic stress states on G_{max} determined for two types of sand. Piezoceramic bender elements were therefore mounted in the top and bottom caps of a conventional Danish Triaxial Apparatus, see Figure 1.

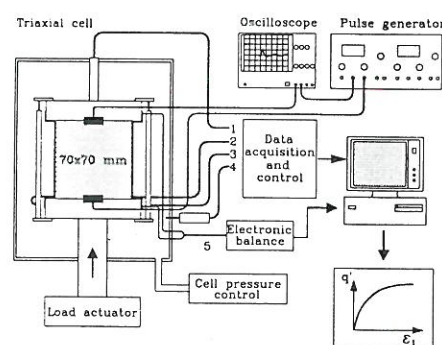


Fig.1. Principle of test set-up.

Different isotropic and anisotropic stress states were applied to the specimens and the travel times for the shear waves were recorded.

In the present study G_{max} was determined for two sands at densities ranging from loose to dense and at different stress states. The results for G_{max} are fitted to expressions describing the variation of G_{max} with stress states and void ratio.

2 MATERIALS AND TEST SET-UP

The tests were performed in the standard Danish Triaxial Apparatus, Jacobsen (1970), fitted with piezoceramic bender elements in the caps, see Figure 1. Two types of sand were used, Baskarp No. 15 and Lund No. 0.

Baskarp No. 15 is a uniform sand from Sweden. The shapes of the largest grains are round, while the smaller grains have sharp edges. The main part of the Baskarp sand is quartz but contains also some feldspar and biotit.

Lund No. 0 is a uniform sand from Denmark. The grains are primarily with sharp edges, because the sand consists of crushed materials. The main part of the sand is quartz.

The classification parameters for the two types of sand are listed in Table 1.

Table 1. Classification parameters for sands.

Parameter	Type of sand	
	Baskarp No. 15	Lund No. 0
e_{max}	0.85	0.82
e_{min}	0.55	0.55
d_s	2.64	2.65
d_{50} [mm]	0.14	0.35
$U=d_{60}/d_{10}$	1.78	2.20

The grain size distribution for the two types of sand is plotted in Figure 2. The percent of grains less than 0.075 mm are 2.0 % and 0.2 % for Baskarp No. 15 and Lund No. 0, respectively.

The piezoceramic bender elements are mounted in the caps used in the Danish Triaxial Apparatus, where arbitrary anisotropic stress states are possible. All the triaxial tests

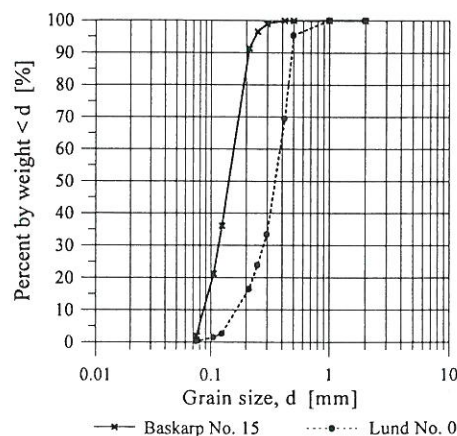


Fig.2. Grain size distribution for Baskarp No. 15 and Lund No. 0.

are carried out with a diameter/height ratio of 1 and smooth pressure heads to obtain homogeneous stress/strain conditions. The initial diameter of the specimen is 70 mm. Radial drainage is used in all the tests.

The test program consists of drained triaxial test carried out on specimens with void ratios varying from minimum to maximum. The different void ratios are obtained by use of the sand rain method, and after placement of the specimens in the triaxial apparatus they are fully saturated by de-aired water. During the consolidation processes performed in the test program, measurements of confining pressure, vertical load, vertical displacement and volume changes are made automatically, inside the cell and as close as possible to the surface of the specimen.

From triaxial test series performed earlier on the two types of sand, strength parameters and parameters to describe the characteristic line defined as $\delta\epsilon_v = 0$, Luong (1982), have been determined.

The void ratios used for the specimens in the current work and the matching strength parameters describing the curved failure line proposed by Jacobsen (1989) are listed in Tables 2 and 3.

The characteristic line is given by $q_{cr} = Mp'$, where q is the deviatoric stress defined as $q = \sigma'_1 - \sigma'_3$ and p' is the mean effective stress defined as $p' = \frac{1}{3}(\sigma'_1 + 2\sigma'_3)$ in the triaxial tests.

The slope of the characteristic line shown in Table 4 for Baskarp No. 0. The characteristic line is independent of void ratio (Ibsen (1993)). All the tests 2, 3 and 4 are determined and carried out in the Danish Soil Mechanics Laboratory, University.

Table 2. Void ratios and parameters for Baskarp No. 15.

Void ratio	ϕ_a [°]
0.55	40.7
0.61	38.4
0.70	34.7
0.85	31.7

Table 3. Void ratios and parameters for Lund No. 0.

Void ratio	ϕ_a [°]
0.55	36.4
0.61	33.4
0.69	30.2

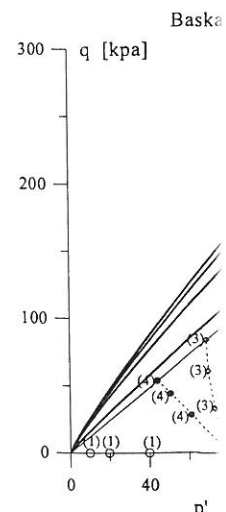


Fig.3. Failure lines (q vs p') for Baskarp No. 15.

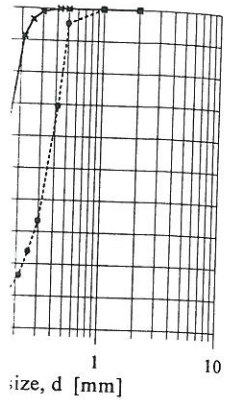


Figure 2.1. Void ratio, e , versus size, d [mm].

meter/height ratio of 1 to obtain homogeneous specimens. The initial diameter is 70 mm. Radial tests.

Tests of drained triaxial specimens with void ratios from 0.09 to maximum. The specimens are obtained by use of the method of placement of the specimen in the apparatus they are submerged in water. During the test, the confining pressure, p' , is increased automatically, inside the specimen, to the surface.

Tests performed earlier to determine strength parameters for the characteristic line (Luong (1982), have

for the specimens in the matching strength parameters (curved failure line (Luong (1989) are listed in

is given by $q_{cl} = Mp'$, where M is the characteristic stress defined as the slope of the characteristic line in the triaxial tests.

The slope of the characteristic line, M , is shown in Table 4 for Baskarp No. 15 and Lund No. 0. The characteristic state is found to be independent of void ratio, Luong (1982) and Ibsen (1993). All the results presented in Table 2, 3 and 4 are determined from triaxial tests carried out in the Danish Triaxial Cell and at the Soil Mechanics Laboratory at Aalborg University.

Table 2. Void ratios and asymptotic strength parameters for the tested specimens on Baskarp No. 15.

Void ratio	ϕ_a [°]	c_a [kPa]	m
0.55	40.7	29	0.13
0.61	38.4	35	0.12
0.70	34.7	31	0.12
0.85	31.7	6	0.09

Table 3. Void ratios and asymptotic strength parameters for the tested specimens on Lund No. 0.

Void ratio	ϕ_a [°]	c_a [kPa]	m
0.55	36.4	48	0.16
0.61	33.4	60	0.16
0.69	30.2	60	0.16

Table 4. Parameters for the characteristic line.

Soil	M
Baskarp No. 15	1.21
Lund No. 0	1.18

For each specimen the travel time for the shear wave is measured at different isotropic and anisotropic stress states. The stress paths applied are shown in Figure 3, all starting from an isotropic stress state at $p' = 80$ kPa:

- (1) isotropic consolidation
- (2) constant mean effective stress, p'
- (3) constant product of $(\sigma'_1)^{0.3}(\sigma'_3)^{0.2}$
- (4) constant σ'_1
- (5) constant σ'_3

The listed stress paths are chosen to facilitate study of the effect on the shear wave velocity/shear modulus of applying shear stresses to the soil specimen.

The stress states with G_{max} measurements are marked with symbols and numbers in Figure 3. Measurements of the travel time of the shear wave are performed at each stress state and after completion of the primary consolidation process to ensure the pore pressure is zero at measuring. To prevent large

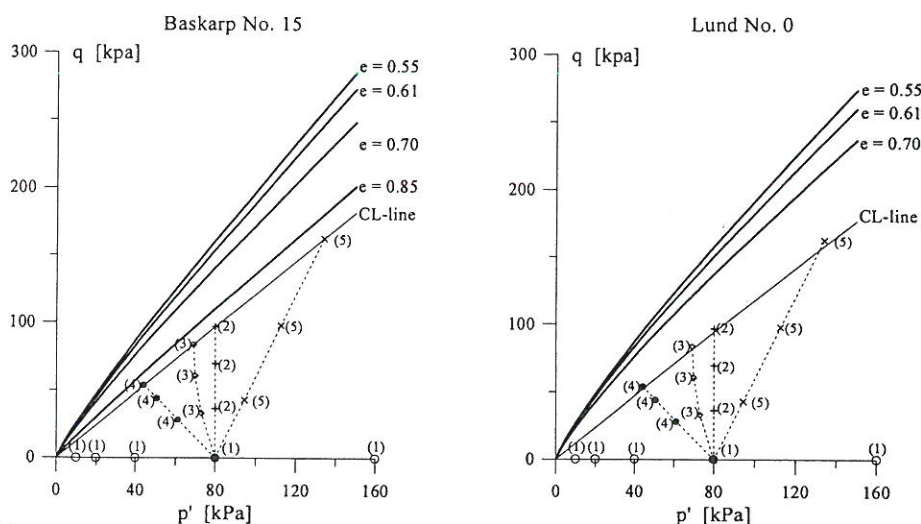


Fig.3. Failure lines (thick lines), characteristic lines (CL-line) and stress paths for the performed test program on Baskarp No. 15 and Lund No. 0.

plastic deformations and therefore large changes in void ratio during a test, all the stress states at measuring are below the CL-line.

The test set-up for the drained triaxial tests are shown schematically in Figure 1. The driving signal for the bender element used as transmitter is a square wave with an amplitude of ± 10 volts.

3 RESULTS

The travel time for the applied shear wave to propagate from one end of the specimen to the other is recorded by an oscilloscope, type HP54654A. A typical trace recorded on the oscilloscope is shown in Figure 4. The square wave driving signal is shown along with the received shear wave. The time difference between sending the square wave and receiving the shear wave is measured directly on the oscilloscope.

The travel time, defined as the time difference between the transmitted shear wave and the first clear inversion of the receiver signal is observed, see Figure 4, along with the travel length for the shear wave give the shear wave velocity as $v_s = l_{eff} / \Delta t$. Here l_{eff} is the tip to tip distance between the elements and Δt is the measured travel time, Dyvik and Madhus (1985) and Viggiani and Atkinson (1995a).

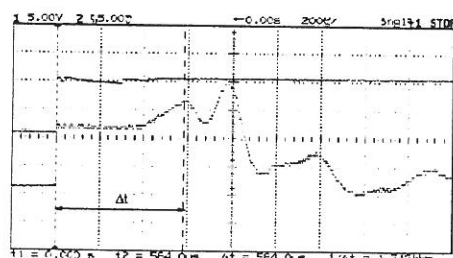


Fig.4. Typical trace for transmitter and receiver signal of bender elements.

Assuming elastic wave propagation the shear modulus, G_{max} , may be calculated as

$$G_{max} = \rho v_s^2 \quad (1)$$

The calculated values of G_{max} by Eq. (1) for the 7 triaxial tests are plotted in Figures 5 and 6. Figure 5 shows the variation of G_{max} with mean effective stress, while Figure 6 shows the variation of G_{max} for the four stress paths, (2), (3), (4) and (5) shown in Figure 3, separately. From Figure 5 it is observed that the shear modulus increases with increasing effective stress level, while it decreases with increasing void ratio. Comparing the stiffness for the two sands, Lund No. 0 has a slightly higher shear modulus than Baskarp No. 15.

All the stress paths start at the isotropic stress state, $p' = 80$ kPa. In Figure 6 G_{max} at the

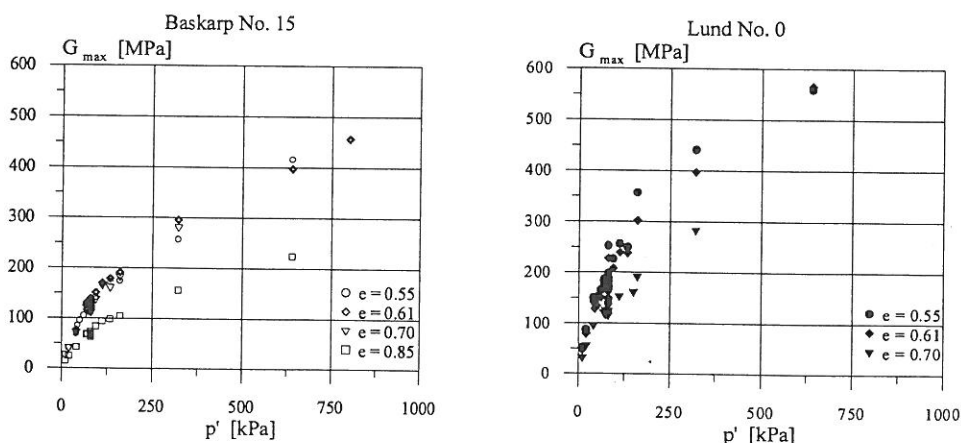


Fig.5. G_{max} determined by bender element tests for Baskarp No. 15 and Lund No. 0 sand at the five stress paths shown in Figure 3.

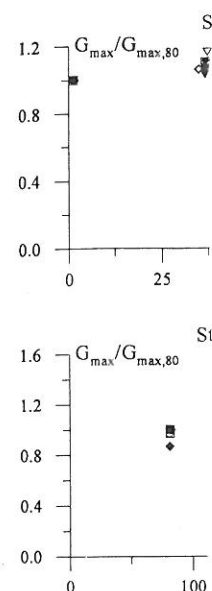


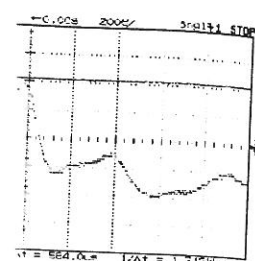
Fig.6. Normalised G_{max} versus p' for Baskarp No. 15 and Lund No. 0 sand.

different stress states at $p' = 80$ kPa are normalized at $p' = 80$ kPa of the current stress path.

From Figure 6 it is observed that the shear modulus levels are identical for the two sands. Comparing the variation of G_{max} for paths (2) and (5) shows that the variation for path (2) is slightly higher than for path (5).

Comparing the variation of G_{max} for paths (3), (4) and (5) shows that the variation for path (3) is slightly higher than for paths (4) and (5). The shear modulus increases slightly with increasing radial stress, σ'_3 , while the shear stress is kept constant.

The findings of Roesler (1985) show that the shear wave velocity, v_s , is independent of the stress normal to the wave propagates.



for transmitter and receiver elements.

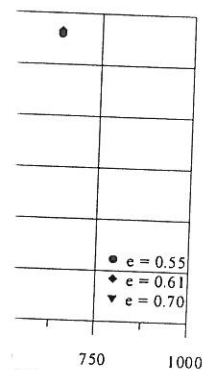
the propagation the shear calculated as

(1)

of G_{max} by Eq. (1) for the data in Figures 5 and 6. The variation of G_{max} with mean normal stress is shown in Figure 6. Figure 6 shows the results for four stress paths, (2), (3), (4) and (5), shown in Figure 3, separately. It is observed that the shear modulus increases with increasing effective stress. The stiffness for the two samples is slightly higher for sample No. 15.

The test results start at the isotropic state. Figure 6 G_{max} at the

0



No. 0 sand at the five

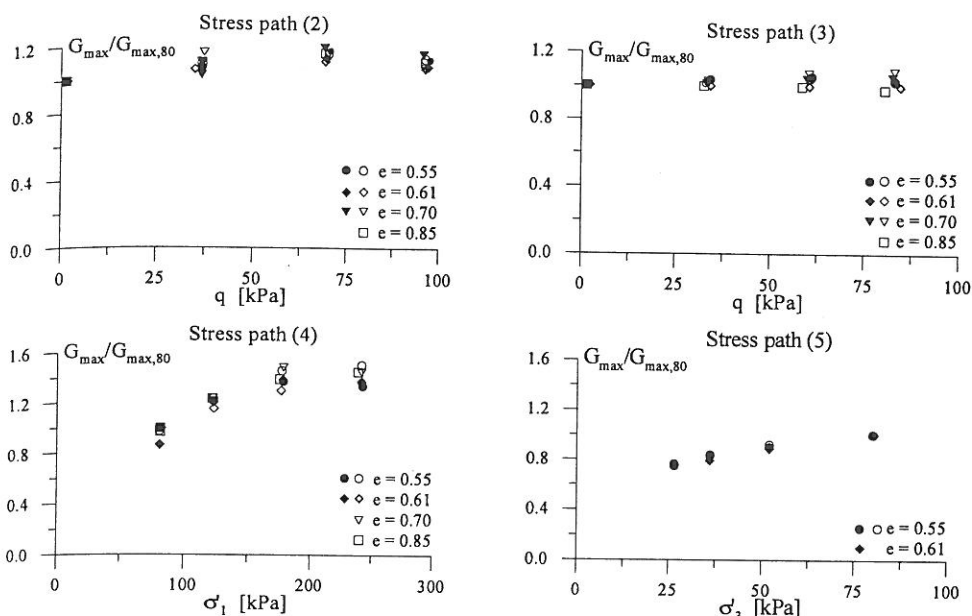


Fig. 6. Normalised shear stress for stress paths (2), (3), (4) and (5) for Baskarp No. 15 and Lund No. 0. The solid symbols correspond to results for Lund No. 0.

different stress states along the four stress paths are normalized with the measured $G_{max,80}$ at p' equal to 80 kPa at the beginning of the current stress path.

From Figure 6 it is seen that the variations in the shear modulus with effective stress levels are identical for the types of sands. Comparing the variation for stress paths (4) and (5) shows that the modulus is a little more dependent on the vertical stress, σ'_1 , than on the radial stress, σ'_3 . The slope of the curve for the variation for path (5) is less than for (4).

Comparing the variations for path (2) and (3), with constant p' and constant product of $\sigma_1'^{0.3}$ and $\sigma_3'^{0.2}$, shows that the shear modulus increases slightly, when the mean effective stress is kept constant but shear stresses are applied, while the change in G_{max} when shear stresses are applied are negligible for path (3).

The findings correspond well to the findings Roesler (1979), who concluded that the shear wave velocity, and therefore the shear modulus, is independent of the effective stress normal to the plane in which the shear wave propagates. A general expression for

G_{max} for frictional materials taking these findings into account is, suggested by Viggiani and Atkinson (1995b)

$$G_{max} = AF(e)p_a^{1-n1-n2}(\sigma'_1)^{n1}(\sigma'_3)^{n2} \quad (2)$$

where A is a parameter depending on the soil characteristics, F(e) describes the dependency of void ratio, p_a is the atmospheric pressure in same units as σ' , $n1$ and $n2$ are elastic constants describing the dependency on stress.

The test results for G_{max} are fitted to eq. (2), where the function $F(e) = 1/(0.3 + 0.7e^2)$ is used, (Hardin and Blandford; 1989). The fitted coefficients for the tests are listed in Table 5.

Table 5. Fitted parameters for Eq. (2).

Soil	n1	n2	A
Baskarp No. 15	0.31	0.26	740
Lund No. 0	0.31	0.26	1065

In Figure 7 the measured values for the shear modulus are compared with the fitted

expressions. In the figure the shear modulus is normalized with the function, $F(e)$.

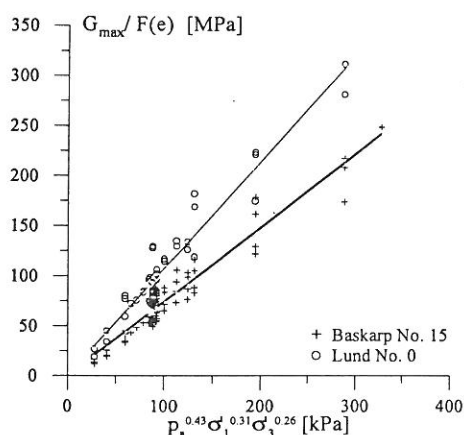


Fig. 7. Comparison between measured values for G_{max} and Eq. (2), $p_a=100$ kPa.

Figure 7 shows that Eq. (2) describes the results from the tests well for the different void ratios. It is also observed that the differences in stiffness between Baskarp sand and Lund sand are described by the factor A in Eq. (2). The characteristics for the two types of soil are very close, but Lund No. 0 contains less grains smaller 0.075 mm than Baskarp. According to Souto et al (1994) the factor decreases with decreasing content of fine grains.

4 CONCLUSIONS

The main factors influencing the shear modulus for frictional soils determined at very low strain level, G_{max} , are primarily the soil characteristics, effective stress level and void ratio.

The shear modulus are not only influenced by the mean effective stress, but also by the shear stress applied. This tendency is observed for both types of sands investigated. However, the dependency of shear stress is much less than of the mean effective stress. It is shown that the effect can be modelled by an expression like Eq. (2).

The dependency of effective stress level is found to be the same for both Baskarp No. 15 and Lund No. 0. The difference in stiffness is

related to the difference in content of fine grains for the sands.

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The anchor caissons and peat layers

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SYNOPSIS: The soil between sand and peat layers was tested by scale pull-out tests. The test results were compared with the polypropylene grid. The efficiency factor, α , was determined. The friction angle and ϕ is determined for the peat layers (e.g. sand and peat). The results cannot be determined by back-calculation because the stress-strain

1. INTRODUCTION

1.1 Modified direct shear tests
The soil-reinforcement has been studied in large shear tests [L×W×H: Fig. 1.

The modified direct shear tests have been carried out in three stages:
1. The textile was fixed against soil into the test box.
2. The textile was fixed against soil into the test box.
3. One end of the reinforcement soil was filled in upper part of the test box.

The modified direct shear tests have been carried out in three stages:

The horizontal displacements were measured. The tests were conducted at a displacement rate dh/dt (%/h).

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